Particle physics: the flavour frontiers

Lecture 6: Flavour and the CKM matrix

Prof. Radoslav Marchevski

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Short recap and today's learning targets

Last time we discussed

- the interactions between the Standard Model fields
- global accidental symmetries of the Standard Model
- count the number of parameters necessary to describe the Standard Model

Today you will …

- be introduced to flavour physics
- analyse the structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix used to describe quark mixing in the Standard Model

Counting the Standard Model parameters

$$N_{\text{phys}} = N_{\text{general}} - N_{\text{broken}}$$

- $\mathcal{L}_{\text{kin}}$: three real parameters, the gauge couplings g, g', g_s
- $\mathcal{L}_{\phi}$: two real parameters v, λ
- $\mathcal{L}_{\text{Yuk}}$(lepton sector): three Yukawa couplings for the leptons y_e, y_μ, y_τ
- $\mathcal{L}_{\text{Yuk}}$(quark sector): six Yukawa couplings for the quarks $y_u, y_c, y_t, y_d, y_s, y_b$, three mixing angles + phase
  - two 3×3 complex Yukawa matrices Y^u, Y^d → 36 parameters (18 real parameters and 18 phases) in a general basis
  - the kinetic terms for the quarks have a global symmetry $G_q = U(3)_Q \times U(3)_U \times U(3)_D$ which has 27 generators
  - the Yukawa terms break the symmetry into a baryon number $H_q = U(1)_B$, which has a single generator → $N_{\text{broken}} = 26$

$$N_{\text{phys}} = 36 - 26 = 10 \Longrightarrow \qquad N_{\text{phys}}^{(r)} = 18 - 9 = 9 \qquad N_{\text{phys}}^{(i)} = 18 - 17 = 1$$

- SM has **18 parameters:** 3 gauge couplings, 2 related to the Higgs potential, 3 charged lepton masses, 6 quark masses, and 4 CKM parameters

Flavour physics

- The appearance of the CKM matrix in the interactions of the W −boson introduces two important ingredients: flavour-changing interactions and CP violation!
- The term flavours is used to describe several mass eigenstates with the same quantum numbers
  - Charged leptons e, μ, τ are in the $(1)_{-1}$ representation
  - Up-type quarks u, c, t are in the $(3)_{+2/3}$ representation
  - Down-type quarks d, s, b are in the $(3)_{-1/3}$ representation
  - Neutrinos ν_1, ν_2, ν_3 in the $(1)_0$ representation
- *Flavour physics:* interactions that distinguish among flavours (W − mediated weak interactions and Yukawa interactions)
- *Flavour parameters:* parameters that carry flavour indices (10 in the SM, 6 quark masses + 4 CKM parameters)
- *Flavour-universal:* couplings are proportional to unit matrix in flavour space (strong, electromagnetic, Z −mediated weak interactions)
- *Flavour-diagonal:* couplings are diagonal but not necessarily universal (Yukawa interactions)

Flavour structure of the Standard Model: mass spectrum

$$-\mathcal{L}_{\text{Yuk}} = \bar{d}_L^i \widehat{M}_d d_R^i + u_L^i \widehat{M}_u u_R^i + \bar{e}_L^i \widehat{M}_e e_R^i + \text{h.c.}, \qquad i = 1, 2, 3$$

$$\widehat{M}_d = \text{diag}(m_d, m_s, m_b)$$

$$\widehat{M}_u = \text{diag}(m_u, m_c, m_t)$$

$$\widehat{M}_e = \text{diag}\left(m_e, m_\mu, m_\tau\right)$$

$$m_f = y_f \frac{v}{\sqrt{2}}$$

Measured masses suggest strong hierarchy between and within families

⇓

Strong hierarchy of the Yukawa couplings

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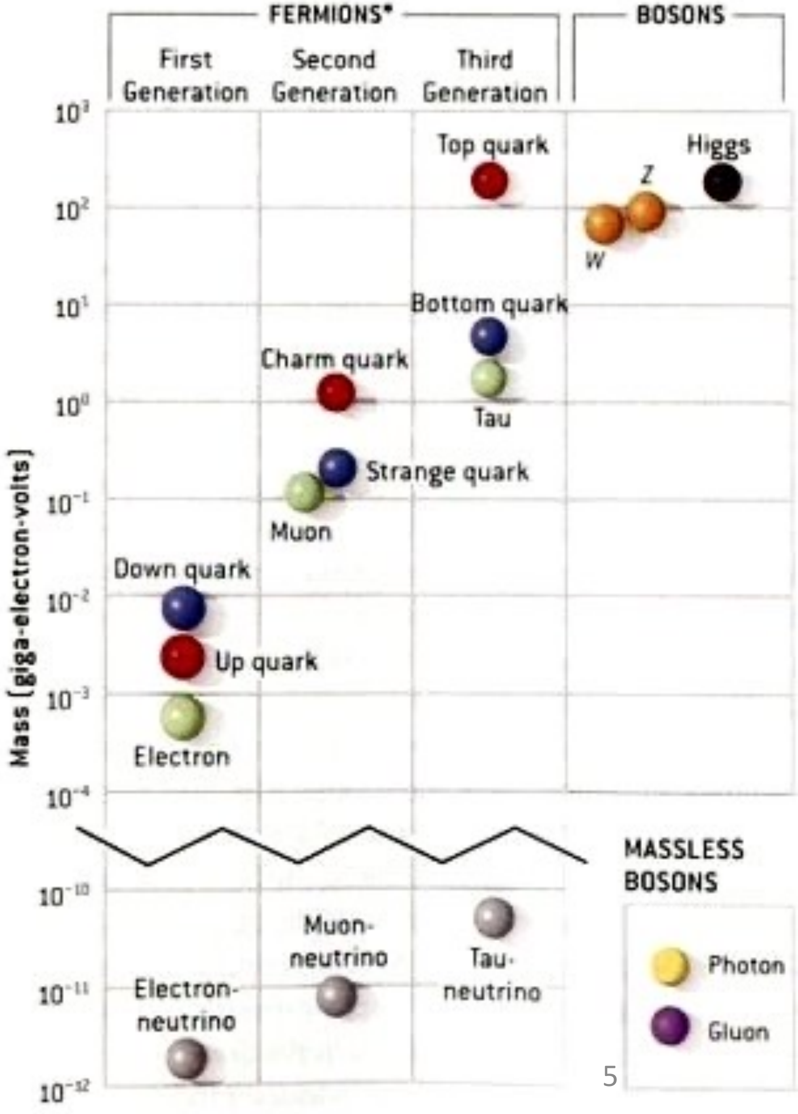
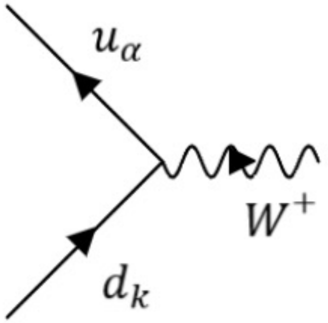
No theoretical reason in the SM



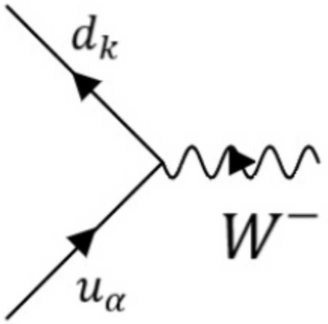
Flavour structure of the Standard Model: cc interactions

$$\mathcal{L}_{W,\,cc} = -\frac{g}{\sqrt{2}}\left[W_\mu^+\left(\bar{u}_L^\alpha V_{\alpha k}\gamma^\mu d_L^k + \bar{\nu}_{eL}^i\gamma^\mu e_L^i\right) + W_\mu^-\left(\bar{d}_L^k V_{k\alpha}^*\gamma^\mu u_L^\alpha + \bar{e}_L^i\gamma^\mu \nu_{eL}^i\right)\right], \qquad \alpha, k, i = 1,2,3$$

Quark current



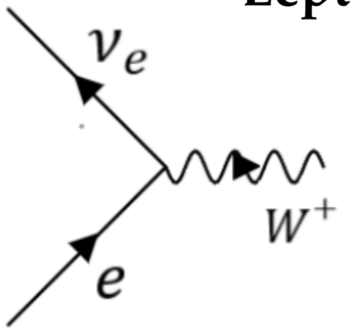
$\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)V_{\alpha k}$ → CKM matrix element



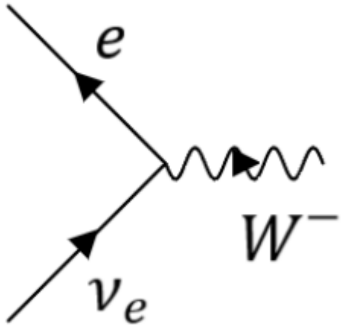
$\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)V_{\alpha k}^*$

can violate flavour (change generation)

Lepton current



$\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$



$\frac{g}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$

cannot violate flavour → lepton flavour universality

CKM matrix

$$\mathcal{L}_{W,cc} = -\frac{g}{\sqrt{2}}\left[W_\mu^+\left(\bar{u}_L^\alpha V_{\alpha k}\gamma^\mu d_L^k + \bar{\nu}_{eL}^i\gamma^\mu e_L^i\right) + W_\mu^-\left(\bar{d}_L^k V_{k\alpha}^*\gamma^\mu u_L^\alpha + \bar{e}_L^i\gamma^\mu \nu_{eL}^i\right)\right], \qquad \alpha, k, i = 1,2,3$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

V_{ud}:
- Superallowed nuclear β decays
- $\pi^+ \to \pi^0 e^+ \nu_e$ decays

V_{us}:
- $K \to l\nu_l$ and $K \to \pi l \nu_l$ decays
- $\tau \to h\nu_\tau$ $(h = \pi, K)$ decays

V_{cd}:
- $D \to \pi l \nu, D \to K l \nu$ decays
- $D^+ \to \mu^+\nu_\mu, D^+ \to \tau^+\nu_\tau$ decays

V_{ub}:
- Inclusive $b \to uX$ measurements
- $B \to \pi l \nu, B \to \rho l \nu, B \to \tau\nu$ decays

V_{cs}:
- $W^\pm \to c\bar{s}(\bar{c}s)$ decays
- $D_s^+ \to \mu^+\nu_\mu, D_s^+ \to \tau^+\nu_\tau$ decays

V_{cb}:
- Inclusive $b \to cX$ measurements
- $B_{(s)} \to D_{(s)}^{(*)} l \nu$ decays

V_{td}, V_{ts}:

<u>Mainly loop processes</u>
- $B - \bar{B}$ oscillations
- $B \to X_s\gamma, B \to \mu^+\mu^-$ decays
- $K^+ \to \pi^+\nu\bar{\nu}$ decays

V_{tb}:
- $\mathcal{B}(t \to Wb)/\mathcal{B}(t \to Wq)$ decays
- $Z \to b\bar{b}$ indirectly via loops

CKM matrix: standard parametrisation

$$\mathcal{L}_{W,\,cc} = -\frac{g}{\sqrt{2}}\left[W_\mu^+\left(\bar{u}_L^\alpha V_{\alpha k}\gamma^\mu d_L^k + \bar{\nu}_{eL}^i\gamma^\mu e_L^i\right) + W_\mu^-\left(\bar{d}_L^k V_{k\alpha}^*\gamma^\mu u_L^\alpha + \bar{e}_L^i\gamma^\mu \nu_{eL}^i\right)\right], \qquad \alpha, k, i = 1, 2, 3$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashyi-Maskawa (CKM) matrix: 3×3 complex unitary matrix

Four physical parameters: three mixing angles + one complex phase

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

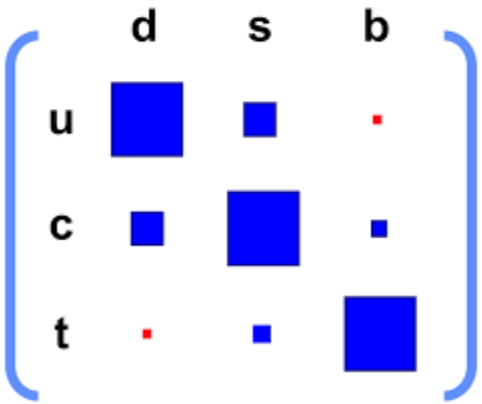
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$s_{ij} = \sin\theta_{ij}$$
$$c_{ij} = \cos\theta_{ij}$$

Standard parametrization used by the PDG (link)

CKM matrix: standard parametrisation

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$



- Experimentally the CKM matrix is very close to a unit matrix
- Strong hierarchy is observed in the off-diagonal elements: $s_{13} \ll s_{23} \ll s_{12} \ll 1$
- We can use an expansion in the small parameter $|V_{us}| = \lambda \approx 0.225$
- Then to an excellent approximation:

$$c_{12} = 1 - \frac{\lambda^2}{2}, \qquad c_{13} = 1, \qquad c_{23} = 1$$

- The virtue of the standard parametrisation is that by measuring $|V_{us}|, |V_{ub}|,$ and $|V_{cb}|$ in tree-level decays one can determine $s_{12}, s_{13},$ and s_{23} simply through

$$s_{12} = |V_{us}|, \qquad s_{13} = |V_{ub}|, \qquad s_{23} = |V_{cb}|$$

CKM matrix: Wolfenstein parametrisation

- The hierarchical structure of the CKM matrix is best represented by the **Wolfenstein parametrisation**

$$V_{us} = s_{12} = \lambda, \qquad V_{cb} = s_{23} = A\lambda^2, \qquad V_{ub} = s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$$

$$\rho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta$$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Doing the change in variables in the standard parametrisation we find a CKM matrix as a function of λ, A, ρ, η that satisfies unitarity exactly!
- Expanding the elements of the standard parametrisation we recover the Wolfenstein matrix and we can find explicit corrections of $O(\lambda^4)$ and higher

CKM matrix: Wolfenstein parametrisation

- The hierarchical structure of the CKM matrix is best represented by the **Wolfenstein parametrisation**

$$V_{us} = s_{12} = \lambda, \qquad V_{cb} = s_{23} = A\lambda^2, \qquad V_{ub} = s_{13}e^{-i\delta} = A\lambda^3(\rho - i\eta)$$

$$\rho = \frac{s_{13}}{s_{12}s_{23}}\cos\delta \qquad \eta = \frac{s_{13}}{s_{12}s_{23}}\sin\delta$$

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\rho = 0.159 \pm 0.010 \qquad \text{A} = 0.826^{+0.018}_{-0.015}$$

$$\eta = 0.348 \pm 0.010 \qquad \lambda = 0.22500 \pm 0.00067$$

Experimentally the CKM is close to a unit matrix

feature of “the” Standard Model and far from “a” generic Standard Model

CKM matrix: $\boldsymbol{CP}$-violation

- Electromagnetic and strong interactions are both invariant under parity P, charge conjugation C, and time reversal T
- Weak interaction violates C, P maximally and the combination CP is violated by the phase δ of the CKM matrix
- We have

$$\bar{u}_L \gamma_\mu d_L \xrightarrow{CP} -\bar{d}_L \gamma^\mu u_L, \qquad W_\mu^{\pm} \xrightarrow{CP} -W^{\mu\,\mp}$$

$$\mathcal{L}_{W,\,cc}^{q} = -\frac{g}{\sqrt{2}}\left[\bar{u}_L^\alpha V_{\alpha k} \gamma^\mu d_L^k W_\mu^+ + \bar{d}_L^k V_{k\alpha}^* \gamma^\mu u_L^\alpha W_\mu^-\right] \xrightarrow{CP} -\frac{g}{\sqrt{2}}\left[\bar{d}_L^k V_{\alpha k} \gamma_\mu u_L^\alpha W^{\mu\,-} + \bar{u}_L^\alpha V_{k\alpha}^* \gamma_\mu d_L^k W^{\mu\,+}\right]$$

The Lagrangian is only invariant under $\boldsymbol{CP}$ if $\boldsymbol{V_{\alpha k} = V_{\alpha k}^*}$ for all $\boldsymbol{\alpha, k = 1, 2, 3}$

At least three generations of quarks are needed to get $\boldsymbol{CP}$ violation

CKM matrix: $\boldsymbol{CP}$-violation

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- The CKM complex phase δ is (the only) source of CP-violation in the SM
- CP-violation in the processes involving quarks: **experimentally well established**
- Experimentally strong hierarchy is observed: $s_{13} \ll s_{23} \ll s_{12} \ll 1$
- Mixing matrix in the lepton sector arising from neutrino mass terms ($d = 5$ operators) COULD lead to CP-violation in lepton processes but **it is not yet observed**
- CP-conservation in QCD is an experimental fact $\Longrightarrow$ **"Strong $\boldsymbol{CP}$ problem"**
  - QCD Lagrangian would require a term that violates $CP \propto \bar{\theta}$ (additional parameter)
  - tight experimental bounds on CP-violation in QCD (no electric dipole moment of the neutron) $\Longrightarrow \bar{\theta} < 10^{-10}$ (Why??)

CKM matrix: Jarlskog invariant

- There is a freedom to define phases
- There are quantities that are invariant under phase rotation (observables)
- *Observables:* $|V_{\alpha i}|^2,\ \ Q_{ijkl} \equiv V_{ij}V_{kl}V_{il}^*V_{kj}^*,\ \arg(Q_{ijkl})$
- In the Standard Model there is one basis-independent invariant, J_{CKM}

$$\mathcal{Im}\left(V_{ij}V_{kl}V_{il}^*V_{kj}^*\right) = \mathcal{Im}\left(Q_{ijkl}\right) = J_{\text{CKM}} \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln}\,, \qquad (i,j,k,l = 1,2,3)$$

- J_{CKM} corresponds to

$$J_{\text{CKM}} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13} \sin\delta \approx \lambda^6 A^2 \eta$$

- The Jarlskog invariant is a very important observable, essential for CP violation and is related to the areas of all CKM unitarity triangles: $A = |J_{\text{CKM}}|/2$

CP violation in "the" Standard Model

- The parameters of the CKM matrix in nature are **far from generic**
- A generic Standard Model violates CP but very specific realisations can conserve CP
- Necessary and sufficient condition for the Standard Model to violate CP:

$$X_{CP} \equiv \Delta m_{tc}^2 \Delta m_{tu}^2 \Delta m_{cu}^2 \Delta m_{bs}^2 \Delta m_{bd}^2 \Delta m_{sd}^2 \, J_{\mathrm{CKM}} \neq 0, \qquad \Delta m_{ij}^2 = m_i^2 - m_j^2$$

- Leading to the following requirements
  - within each quark sector there must be no mass degeneracy
  - the Jarlskog invariant must not vanish

CKM matrix: unitarity relations

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \qquad VV^{\dagger} = V^{\dagger}V \quad = \mathbb{I} \text{ (unitarity)}$$

- Unitarity condition → orthogonality
- Unitarity leads to the following set of equations (normalization of the columns and rows of the CKM matrix)

$$V_{ij}V_{kj}^{*}(i = k = u, c, t) = 1 \qquad V_{ij}V_{ik}^{*}(j = k = d, s, b) = 1$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1 \propto O(1) + O(\lambda^2) + O(\lambda^6) \qquad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \propto 1 + O(\lambda^2) + O(\lambda^6)$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1 \propto O(\lambda^2) + O(1) + O(\lambda^4) \qquad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 \propto O(\lambda^2) + O(1) + O(\lambda^4)$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1 \propto O(\lambda^6) + O(\lambda^4) + O(1) \qquad |V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1 \propto O(\lambda^6) + O(\lambda^4) + O(1)$$

CKM matrix: unitarity triangles

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \qquad VV^\dagger = V^\dagger V \;= \mathbb{I}\ \text{(unitarity)}$$

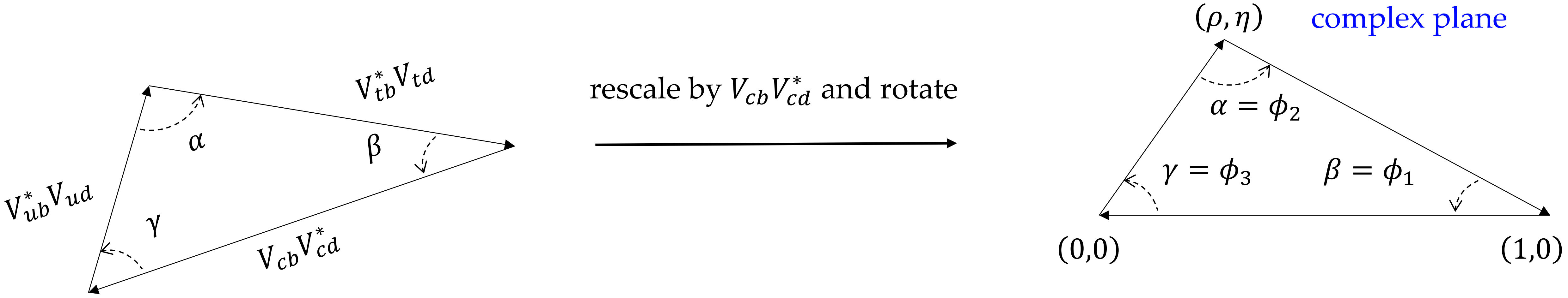
- Observables (invariant under phase transformations): $|V_{\alpha i}|^2$, $Q_{ijkl} \equiv V_{ij}V_{kl}V_{il}^*V_{kj}^*$, $\arg(Q_{ijkl})$
- Unitarity condition → orthogonality
- Geometrical interpretation of the off-diagonal elements: 6 independent “unitarity” triangles

$$\sum_{i=u,c,t} V_{iq}V_{iq'}^* = 0, \qquad (qq' = ds, db, sb) \qquad\qquad \sum_{i=d,s,b} V_{qi}V_{q'i}^* = 0, \qquad (qq' = uc, ut, ct)$$

"The" unitarity triangle

- Geometrical presentation of one of the triangles

$$\sum_{i=u} V_{iq} V^*_{iq'} = 0, \qquad (qq' = db) \qquad \Longrightarrow \qquad V^*_{ub} V_{ud} + V^*_{tb} V_{td} + V_{cb} V^*_{cd} = 0$$



$$\alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) = \arg(-Q_{ubtd})$$
$$\beta = \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) = \arg(-Q_{tbcd})$$
$$\gamma = \arg\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right) = \arg(-Q_{cbud})$$

observables

$$A = \frac{1}{2}[V_{cd}V_{cb}][V_{ud}V_{ub}\sin\gamma] = \frac{1}{2}|Q_{udcb}\sin\gamma| = \frac{1}{2}|\mathcal{J}m(Q_{udcb})| = \frac{1}{2}|J_{\rm CKM}|$$

$$J_{\rm CKM} = c_{12}c_{23}c^2_{13}s_{12}s_{23}s_{13}\sin\delta \approx \lambda^6 A^2 \eta$$

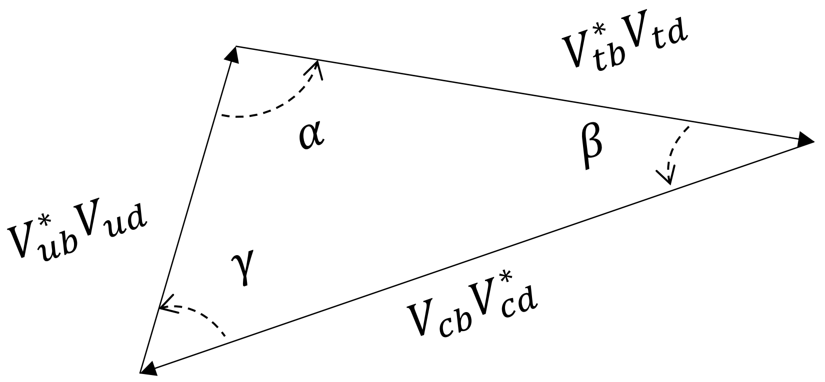
$$|J_{\rm CKM}| < \frac{1}{6\sqrt{3}} \sim 0.1$$

Best fit: $J_{CKM} = \left(3.115^{+0.047}_{-0.059}\right) \times 10^{-5}$

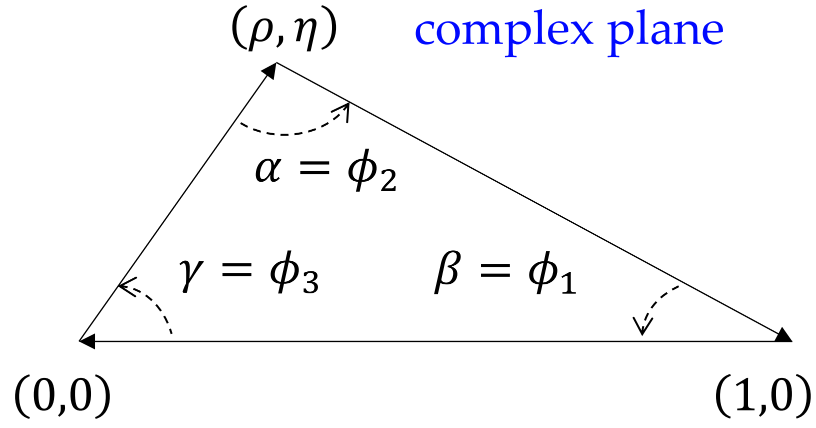
“The” unitarity triangle

- Geometrical presentation of one of the triangles

$$\sum_{i=u} V_{iq} V^*_{iq'} = 0, \qquad (qq' = db) \qquad \Longrightarrow \qquad V^*_{ub} V_{ud} + V^*_{tb} V_{td} + V_{cb} V^*_{cd} = 0$$



rescale by $V_{cb}V^*_{cd}$ and rotate



$$\alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right) = \arg(-Q_{ubtd})$$
$$\beta = \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right) = \arg(-Q_{tbcd})$$
$$\gamma = \arg\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right) = \arg(-Q_{cbud})$$

observables

$$A = \frac{1}{2}[V_{cd}V_{cb}][V_{ud}V_{ub}\sin\gamma] = \frac{1}{2}|Q_{udcb}\sin\gamma| = \frac{1}{2}|\mathcal{Im}(Q_{udcb})| = \frac{1}{2}|J_{\text{CKM}}|$$

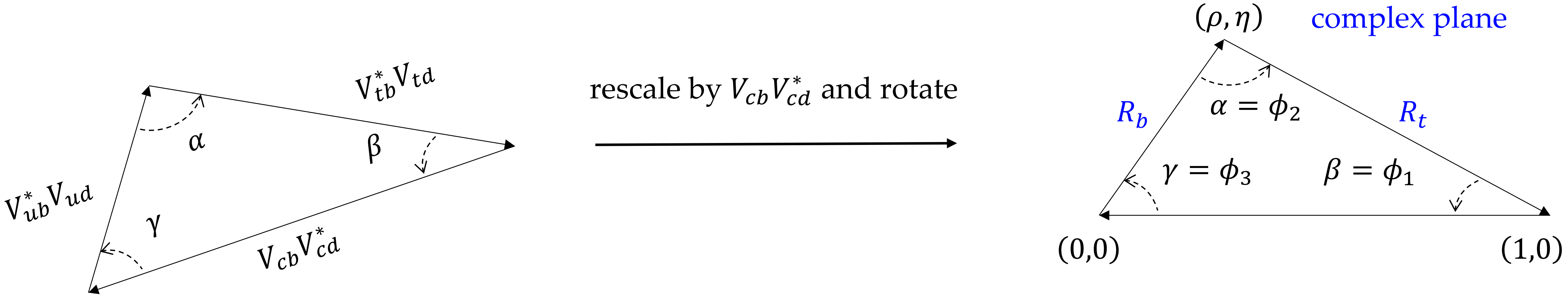
All unitarity triangles have equal area - $|J_{\text{CKM}}|/2$

CP-violation only if $J \neq 0$

"The" unitarity triangle

- Geometrical presentation of one of the triangles

$$\sum_{i=u} V_{iq} V^*_{iq'} = 0, \qquad (qq' = db) \qquad \Longrightarrow \qquad V^*_{ub} V_{ud} + V^*_{tb} V_{td} + V_{cb} V^*_{cd} = 0$$



$$R_t = \left|\frac{V_{td} V^*_{tb}}{V_{cd} V^*_{cb}}\right| = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$$

$$R_b = \left|\frac{V_{ud} V^*_{ub}}{V_{cd} V^*_{cb}}\right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

} observables

$$V_{td} = |V_{td}| e^{-i\beta}, \qquad V_{ub} = |V_{ub}| e^{-i\gamma}$$

$$\alpha + \beta + \gamma = 180^\circ \text{ (unitarity)}$$

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$

Goal of unitarity triangle tests

Basic idea: measure the 4 CKM parameters in many different ways

- Over-constrain the triangle by making measurements of all parameters and comparing their consistency
- Particularly useful are the comparisons of measurements of the same parameters in tree-level processes (pure SM) and those made with loops (more sensitive to New Physics)
- Any inconsistency is a signal of New Physics!

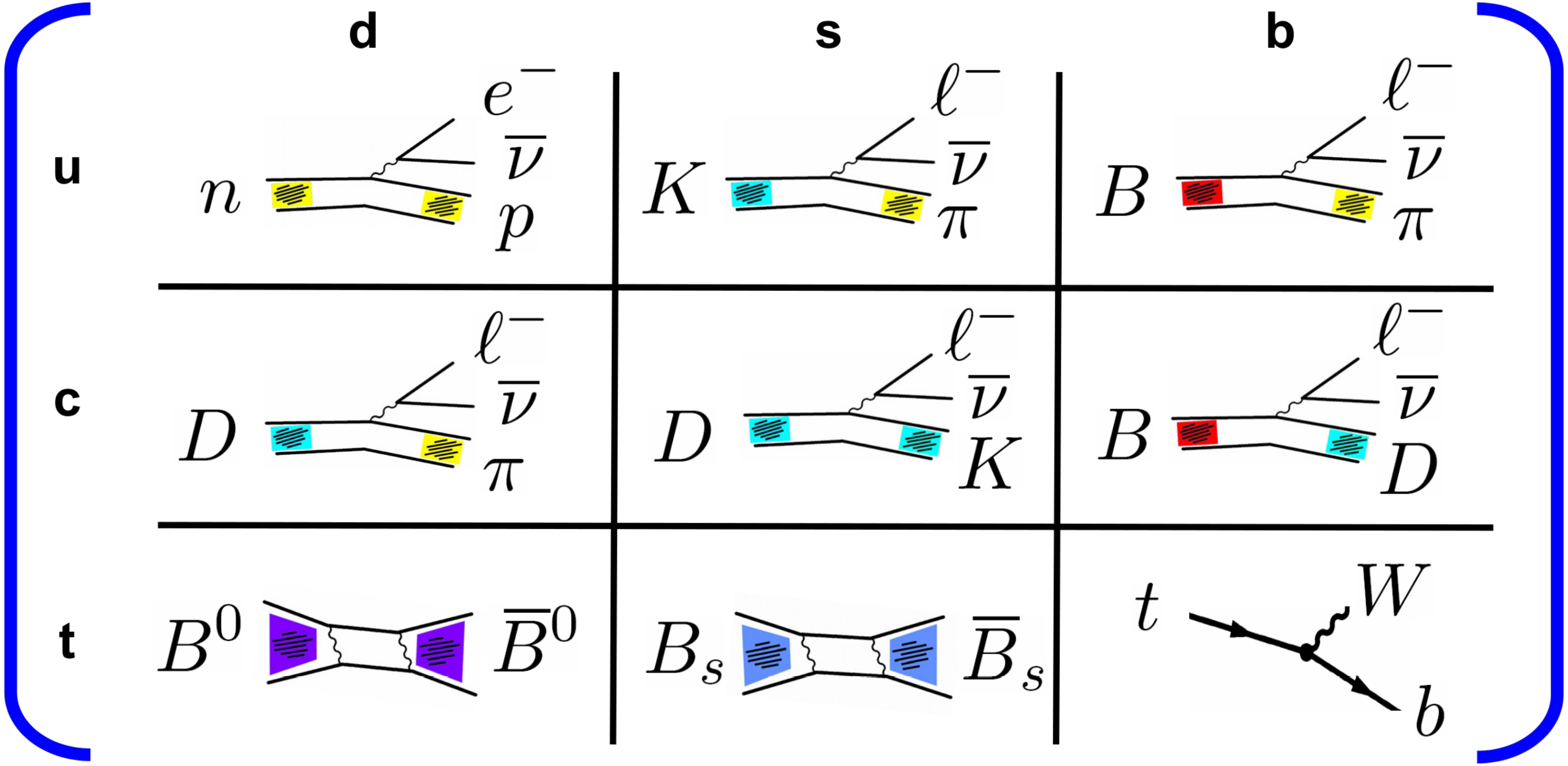
Problems: experimental errors and theoretical errors

- We have to be smart …
    - smart theory to reduce errors
    - smart experiment to reduce errors
- There are cases where both errors are very small (sweet spot!)

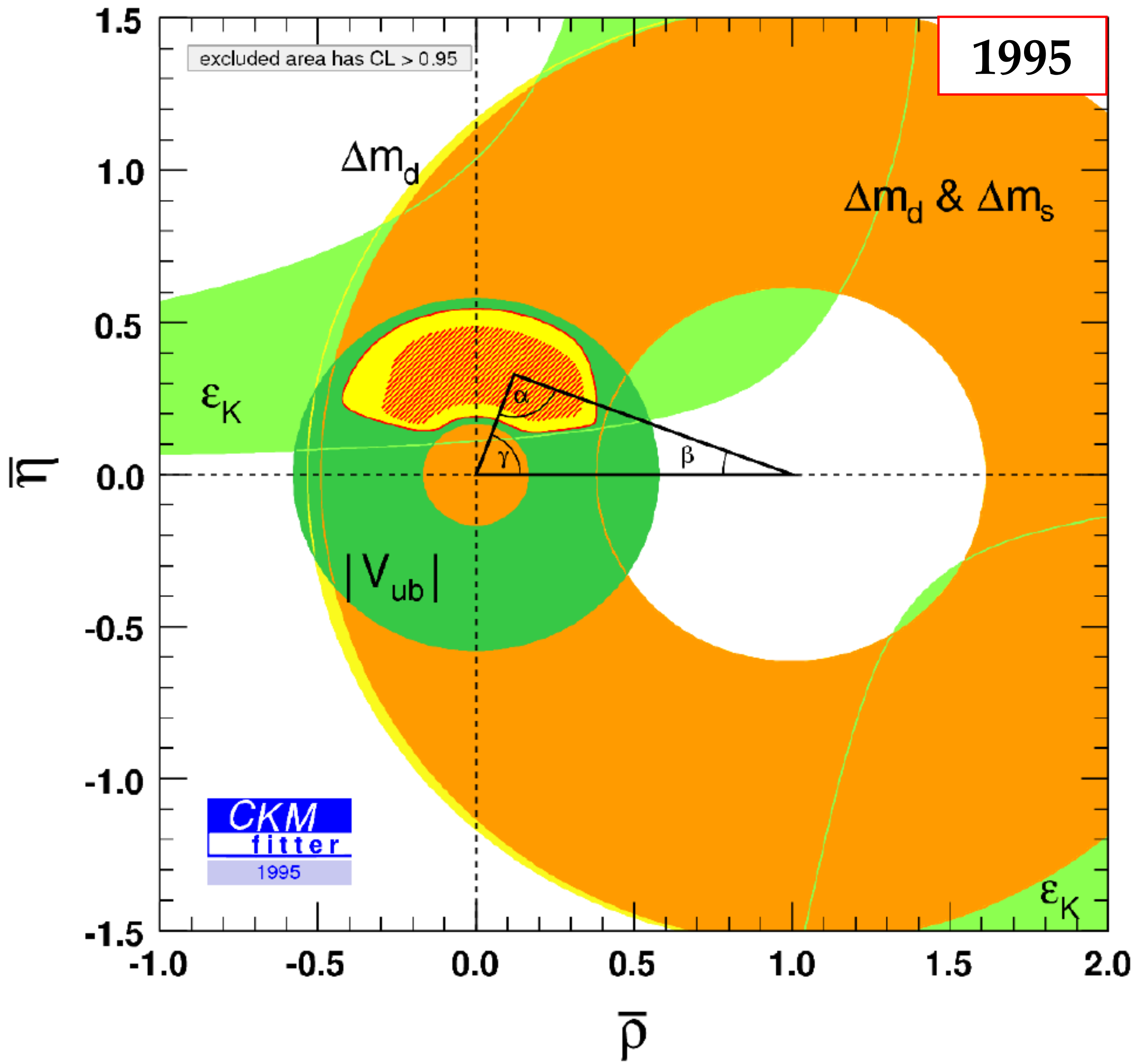
CKM matrix summary

$$\mathcal{L}_{W,cc} = -\frac{g}{\sqrt{2}}\left[W_\mu^+\left(\bar{u}_L^\alpha V_{\alpha k}\gamma^\mu d_L^k + \bar{\nu}_{eL}^i\gamma^\mu e_L^i\right) + W_\mu^-\left(\bar{d}_L^k V_{k\alpha}^*\gamma^\mu u_L^\alpha + \bar{e}_L^i\gamma^\mu \nu_{eL}^i\right)\right], \qquad \alpha, k, i = 1, 2, 3$$

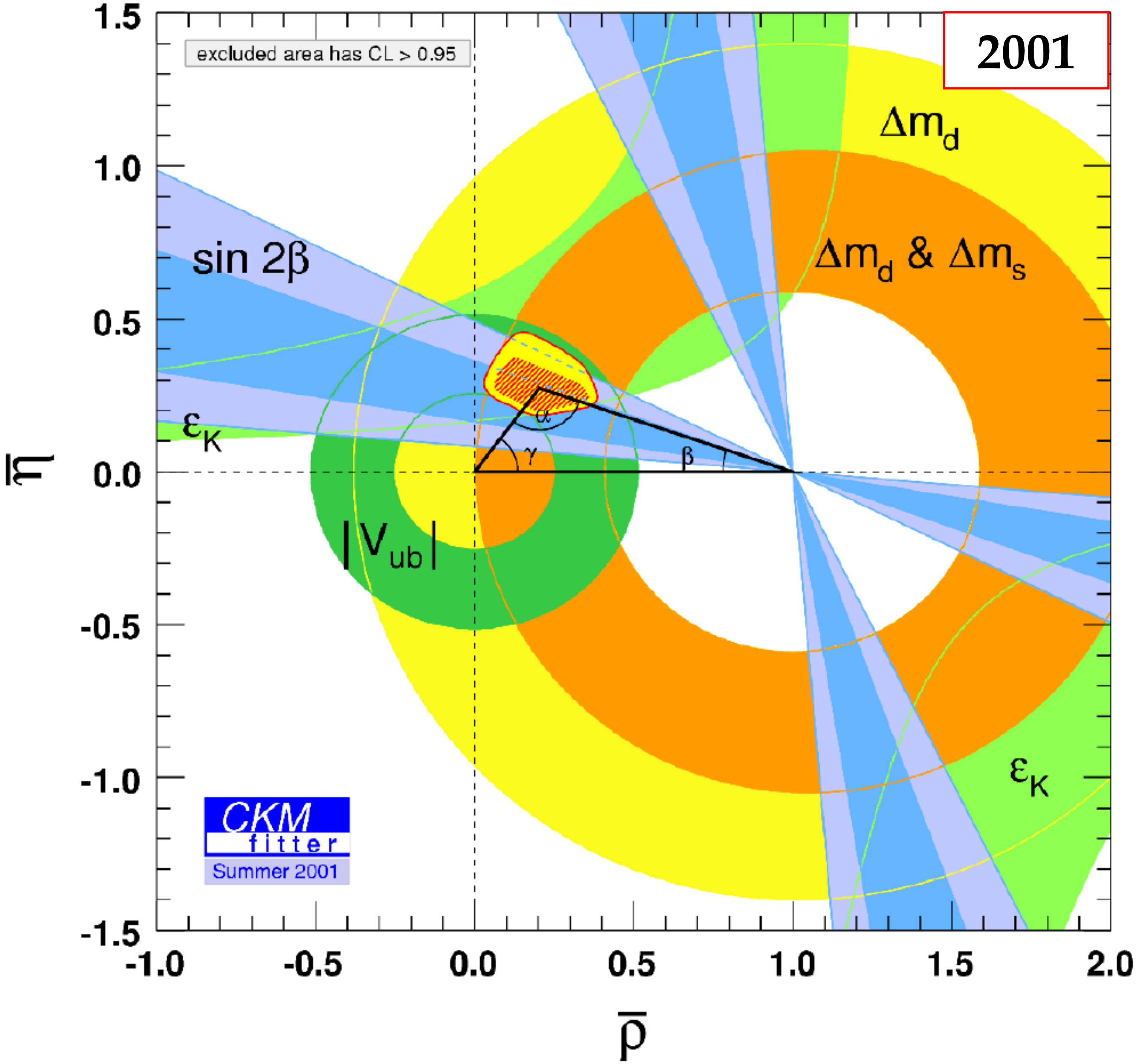
V =



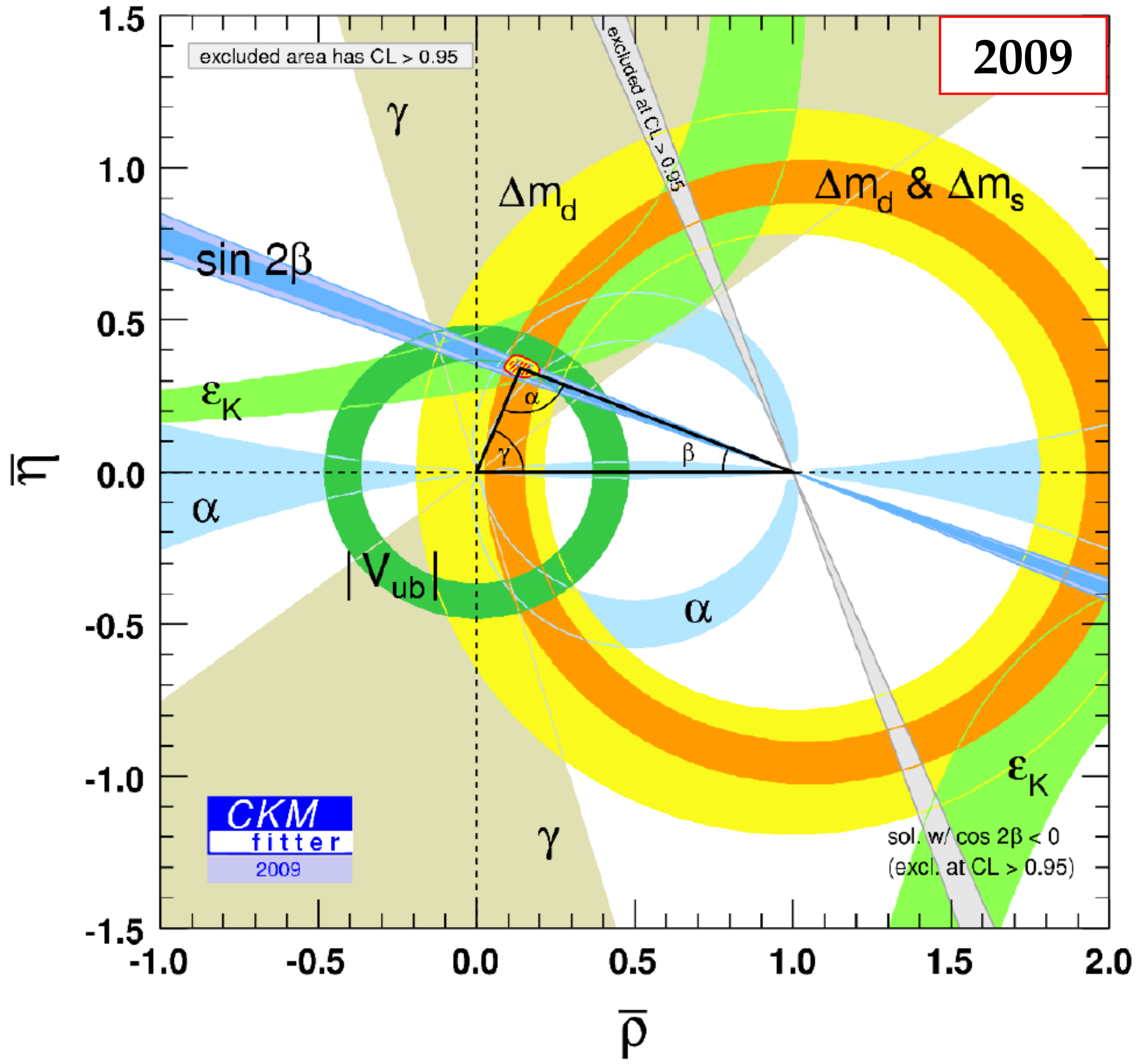
Unitarity triangle: ~ 30 years of progress



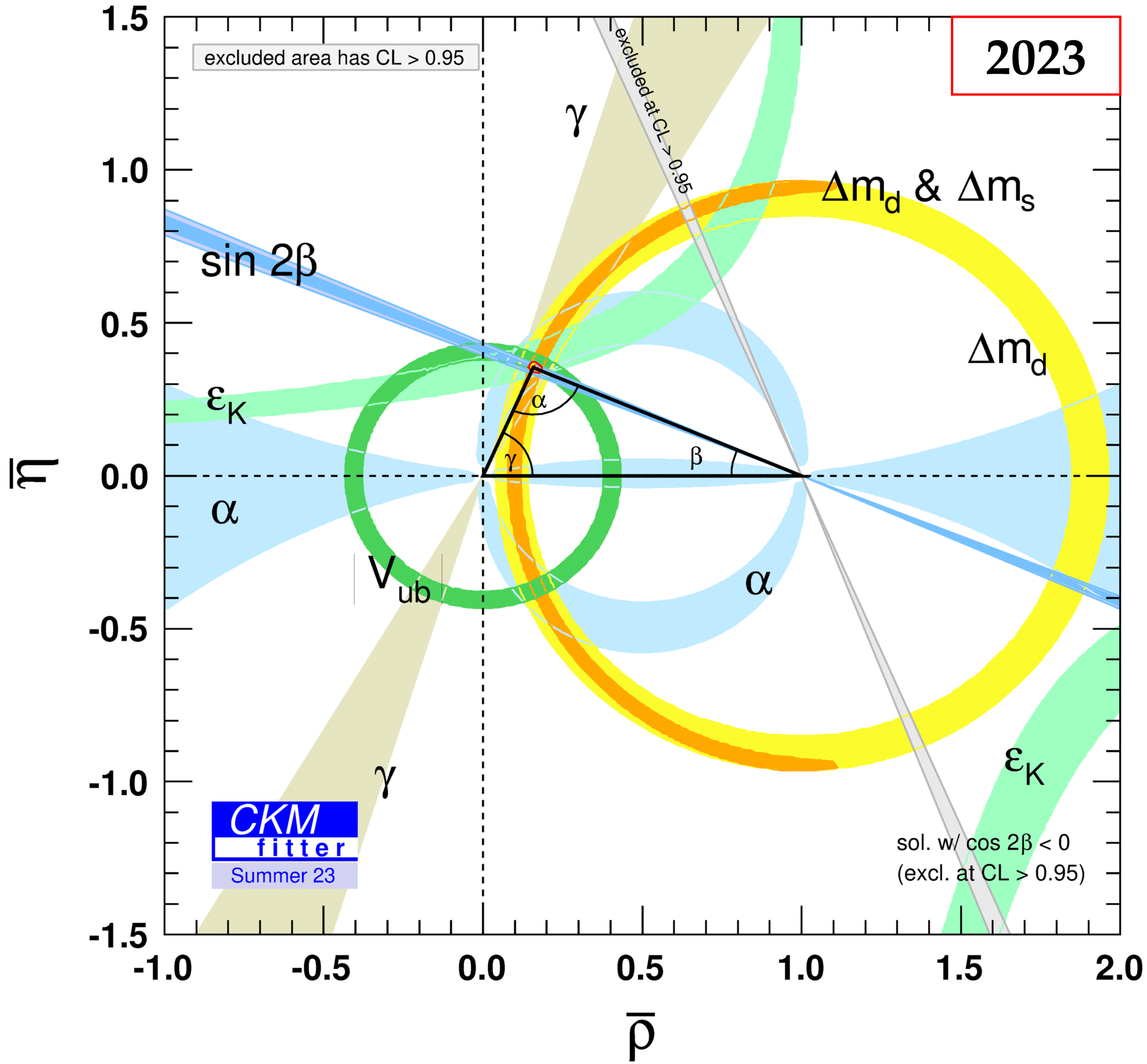
Unitarity triangle: ~ 30 years of progress



Unitarity triangle: ~ 30 years of progress

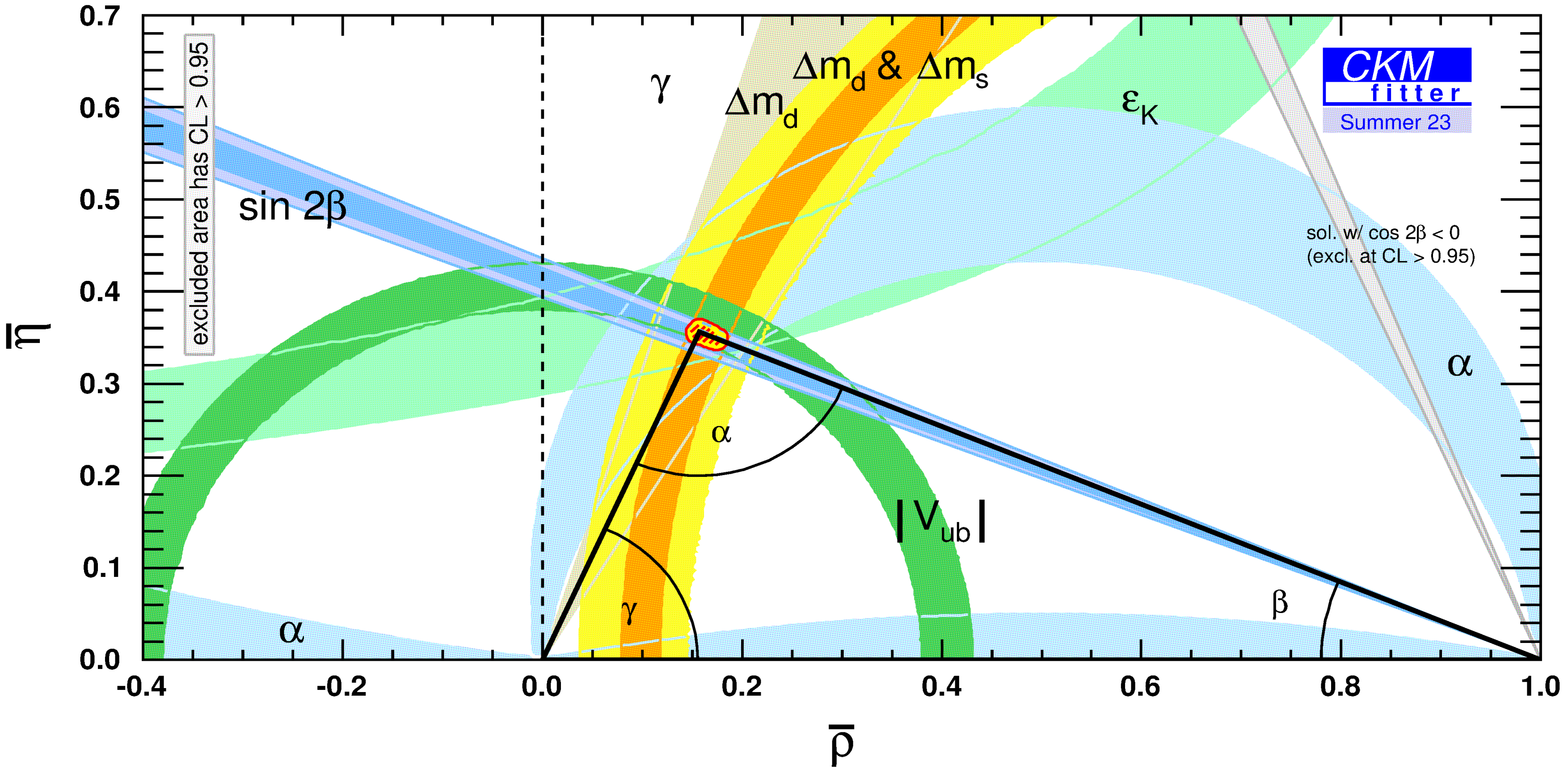


Unitarity triangle: ~ 30 years of progress



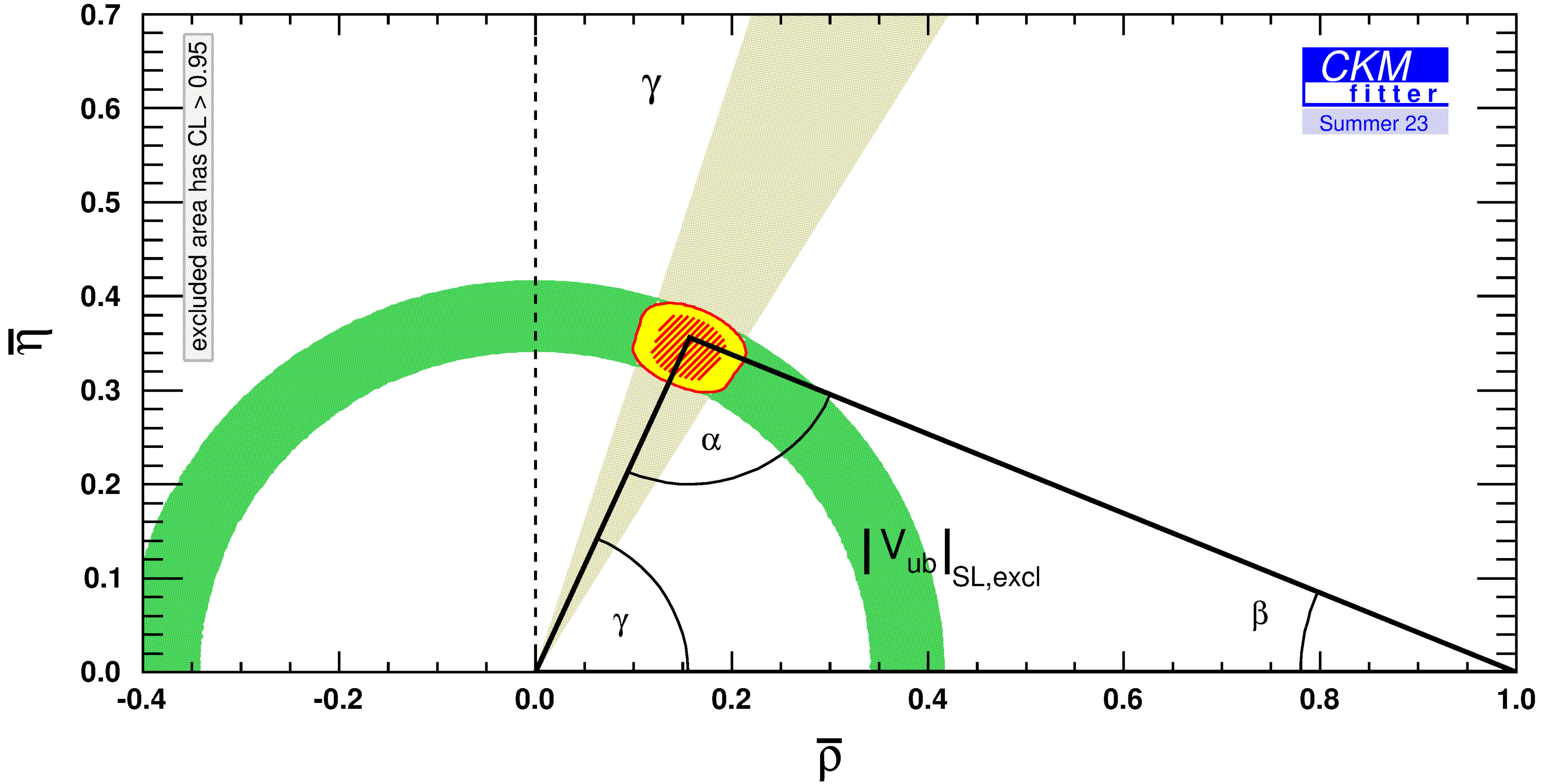
Unitarity triangle: ~ 30 years of progress

- Broad consistency between all current measurements of the UT
- *The CKM paradigm:* dominant mechanism of CP-violation in nature
- *However*, certainly possible for New Physics to give ~10% level effects
- **We need more measurements!**



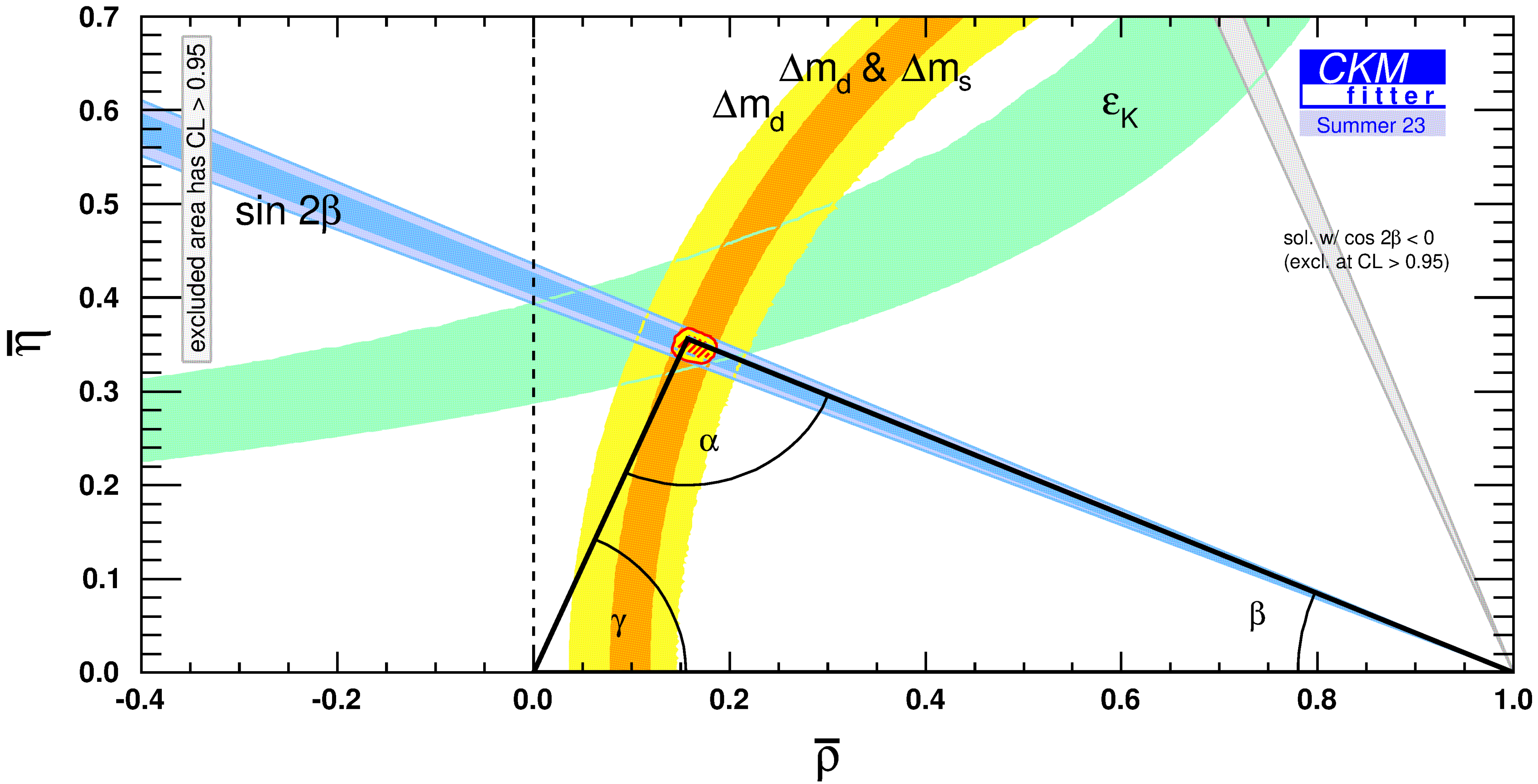
Unitarity triangle: tree level

- Unitarity triangle formed from only tree-level quantities → assumed pure SM
- Tree-level observables are γ and the $|V_{ub}|/|V_{cb}|$ side



Unitarity triangle: loop level

- Unitarity triangle formed from only loop-level quantities → possibility of NP effects
- There is a good consistency between the tree and loop measurements
- Need to improve the precision of tree-level processes to allow for a more sensitive comparison



Summary of Lecture 6

Main learning outcomes

- Introduction to the flavour structure of the SM: *mass spectrum, flavour changing interactions*
- The quark-mixing CKM matrix
    - what parametrisations are commonly used in particle physics
    - how does CP violation arise in the SM
    - how imposing unitarity on the CKM matrix allows us to construct unitarity triangles
    - experimental tests and constraints